Invited Lecture

On the Notion of Mathematical Competence

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ABSTRACT This contribution analyzes the origin of the competence construct, its evolution and how it is conceptualized by different authors in different fields. The objective is to reveal the complexity of the idea that the construct is meant to capture; in fact, only by bringing out this complexity can we hope to make the construct truly operational and useful for practice and educational research. In particular, I discuss the multidimensional artefact-like character of the construct of competence trying to reveal the several distinct related dimensions which contribute to form this single theoretical concept.

Keywords: Competence; Mathematical competence; Multidimensional construct.

1. Introduction

In this contribution I discuss the complexity of the idea of competence and of mathematical competence at two distinct levels:

- At a general level, the discussion focuses on how the idea of competence is conceptualized in different domains and from different subjects; the interest is on the variety of ways in which the idea is conceptualized within and outside mathematics education, and on the different elements that these conceptualizations bring to light.
- And at a more particular level, the discussion focuses on a specific elaboration of the idea of competence in education, on its relevance for mathematics education, on the features these conceptualization helps to grasp and its intrinsic complexity.

The discussion also concerns the issue of the objectives, for which one needs or wants to define the idea of competence. This leads to reflect on the *artefact-like nature* of the notion of competence. These aspects are intertwined with each other, and emerge together throughout the discussion.

The structure of the contribution is the following. First of all, I will try to make clear the reasons for my interest in the idea of competence (section 2), and introduce the issue of its elusiveness (section 3) which will be further developed in the following sections. Then, I will outline the origin and development of the idea of competence in the last decades (section 4) and examine the idea as it emerges from literature in social and

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behavioral sciences (section 5). To this end, I will refer to the surveys by Weinert (2001) and Mulder (2017). I will present some constructs in mathematics education, which can be related to the idea of competence (section 6). These constructs resonate at different levels with a specific definition of the idea of competence elaborated in the field of education. Drawing on this definition and I will discuss the various critical issues that characterize in an intrinsic, unavoidable manner the idea of competence (section 7). I will then try to trace some conclusive reflections on this theme (section 8).

1. The Interest for The Idea of Competence

The idea of competence has acquired great importance in recent years in the context of the debate on education and training promoted in Europe (and consequently in Italy, where I come from) following the so-called Lisbon strategy. This debate has been prompted in by diverse Political institutions (e.g. in Europe by the European Parliament and Council, in the context of the so-called Lisbon strategy) but also economical organizations (such as OECD), which defend the adoption of the term, and the perspective it conveys, in order to highlight the importance of an approach to teaching and learning not bound to specific discipline content knowledge.

However, the interest in the idea of competence is not restricted to the European context or to the debate on the Lisbon strategy in Europe. In fact, the discourse on the idea of competence involves various fields of social sciences, psychology, and educational sciences, with particular reference to the world of work, to incoming and ongoing vocational training, to professional qualification, to mobility ... But the term is also used outside the purely professional context.

All that does not remain at a declarative level, but has practical consequences, in that curriculum and assessment are now expected to be framed by this perspective. As a consequence, teachers have to cope with new professional challenges concerning the design of educational activities and assessment of students" learning by focusing on competence development.

2. An Elusive Idea

As mentioned above, the idea of competence is at the intersection of different fields and research domains and it is used by different actors — researchers, teachers, policy-makers, various stakeholders — within different communities and with different aims. But contributions from different fields are not always clearly connected to each other.

In such a situation, it does not help that the idea of competence is rarely defined — its meaning is assumed to be clear or intuitive — or it is defined, maybe inevitably, by quasi-synonyms: capacity, ability, proficiency… (Kilpatrick, 2020). The idea of competence, then, appears to one of the most elusive in education (Kilpatrick, 2020). This same view has been expressed over the years by several authors. For instance, Dolz and Ollagnier in the early 2000s edited a book titled "the enigma of competence in education" (2002, my translation); Gilbert and Parlier (1992) used the metaphor of the *sponge-word* to refer to the term competence: as a sponge, it gradually absorbs all

the meanings attributed to it by those who employ it, but, when pressed, it empties and does not reinstitute any meaning. Although Gilbert and Parlier wrote in 1992, the danger they point to is still present.

In next sections, 4 and 5, I analyze the origin of the idea of competence, its evolution and how it is conceptualized by different authors in different fields. This analysis is important, on the one hand because it contribute to detect the complexity of the idea that the construct of competence is meant to capture. On the other hand, because only by bringing this complexity to the light can we hope to make the construct truly operational and useful for educational practice and research. Furthermore, such reflection is needed to make more fruitful the discussion between the different communities that revolve around the world of education. The extensive use of the term does not suffice in itself to assure that there is a common shared perspective amongst the various stakeholders.

3. The Evolution of The Idea of Competence in Education

The notion of competence has distant roots: the term derives from Latin *competens* and *competentia*, the meaning of which are close to the current common-sense meanings, but the origin of the idea can be traced back to Greek philosophy (Mulder et al., 2007; Pellerey, 2013). Beyond these ancient roots, it is difficult to clearly trace the evolution of the idea: different authors have identified various, apparently independent origins of the emergence and diffusion of the idea of competence in different fields in the last decades.

In the educational field, according to Pellerey (2004), the idea of competence begins to be explored around the 1960s, as a means to describe the expected outcomes of teaching interventions. At first, outcomes are described in terms of "final observable and, in some way, measurable behaviours" (ibidem, p.35, my translation). This initial approach reveals the dominant behavioural approach of those years, characterized by the identification of the idea of competence with that of performance, which pervades the school context for a long time. Later on, the notion of competence is progressively enriched thanks to the reflection developed in the world of work since the end of the seventies. From those years we witness the so-called "de-taylorization" of work (Terraneo and Avvanzino, 2006), which leads to a radical change of paradigm in the conception of the relationship between work and production, and of the organization of work itself. In a Taylor-like work organization, the activity, that the worker must perform, is broken down into simple units, prescribed operations, predefined in a complete and detailed manner. When the organization of work must appeal to the initiative and versatility of the workers, it is necessary for them to develop and be able to mobilize competences related to facing the unexpected, innovating or deciding in uncertain situations (ibidem, p.17). This new paradigm leads to rethinking professional training and, with it, education in general. In this context, McClelland's contribution appears particularly relevant (1973, cited in Pellerey, 2004; in Mulder, 2007; and in

Mulder et al., 2007), in that launches a series of studies aimed at promoting competences as a tool for personnel selection.

In previous years, Chomsky (1968) introduces, in linguistics, the distinction between performance and competence. Even if the meaning that Chomsky attributes to the term "competence" differs from those generally assumed in the educational field, and even if his influence on educational research will be felt only later (Pellerey, 2004), nevertheless this distinction is of crucial importance. Competence becomes therefore conceived as the abstract capacity possessed by an individual, while performance is considered as the possible manifestation of a competence. Likewise, it is recognized that the quality of performance does not depend only on the set of knowledge and skills that the individual may or may not possess, but it is based on a number of factors that are not directly observable.

This brief overview gives an idea of the rich elaboration that took place around this idea, which over the years has been influenced by different theoretical approaches and paradigms. Despite this heterogeneity, general trends can be traced in the development of the idea of competence. Based on the analysis of Mulder, Weigel and Collins (2007), Marzano and Iannotta (2015) identify three main directions along which the development of the notion of competence took place.

- "*From simple to complex*. Competence is an improvement of the knowledge already owned by subject that involves the activation of knowledge, skills and dispositions. The process engages the cognitive, the motivational and the emotional dimension.
- *From the outside to the inside.* According to this process, knowledge draws attention to all those subjective dimensions that are not directly observable outside, but that form the basis of individual behavior.
- *From theoretical to pragmatic.* Competence is specifically assumed and it is related to a given context, losing its general sense. Competence is identified with the subject's ability to use operational strategies for the solution of the problem related to specific culture and contextual dimension" (ibidem, p.10).

4. The Idea of Competence in Social and Behavioural Sciences

From the reconstruction of the historical development of the idea of competence presented in the previous section, a common general understanding of the idea emerges. In fact, there emerges a shared attempt to characterize competence as a system of "prerequisites" or "conditions" necessary to undertake effective actions in the context of certain activities, "[A]a set of capabilities […] which are necessary conditions for effective performance" (Mulder, 2017, p. 1079). However, as we have already mentioned, there are significant differences in the landscape of social and behavioral sciences. To get an idea of this variety we can refer to the analyses of Weinert (2001), and Mulder (2017).

The former investigates the ways in which competence has been defined, described, or interpreted theoretically — in social and behavioral sciences and identifies nine different theoretical approaches to the notion of competence.

The latter tries to extrapolate from the definitions used in the literature (in particular, with reference to vocational training) the main components that can be involved in the definition of the competence construct, and clarify them.

In both, the focus is ultimately on competence as an attribute of the individual.

4.1. A survey on the idea of competence

In his survey Weinert (2001) identified 9 different approaches to the idea of competence, I mention only some of them to give the flavour of the great variety of conceptualizations Weinert recognized:

- System of *general cognitive resources* independent of the content and context of the activity — e.g. working memory, processing speed
- System of *highly specialized knowledge, skills, routines* ... which depend on the content or context of the activity — e.g. chess playing, piano playing, automobile driving, mathematical problem solving, trouble-shooting in complex systems
- System of *cognitive resources and motivational action tendencies*, including factors such as motivation, sense of self, sense of self-efficacy, belief systems.
- *Metacognitive resource system*, either declarative or pragmatic, concerning the management and regulation of one's own cognitive resources; the ability to use knowledge about own knowledge.
- System of *key-competencies*.

With respect to these multiple meanings Weinert (2001) observes that "Unless one argues that the individual prerequisites for the array of cognitive performances and goal directed actions must include all primary mental abilities, all learned skills, knowledge and strategies, the entire complex of learning and achievement motives, and all important vocational skills, the various definitions of competence listed […] are mutually exclusive on a phenomenological, conceptual or theoretical level".

4.2. The several dimensions implied in the idea of competence

Mulder analysed the definitions used in the literature, in vocational training, and extrapolated the various dimensions involved in those definitions. Several different dimensions emerge from this survey: *contextuality*, *developability*, *measurability*, *definability*, *centrality*, *knowledge inclusion*, *dynamic nature*, *mastery level*, *performativity*, and *transferability*.

In order to illustrate these dimensions, I invite readers to consider the following issues, and ask themselves whether or to what extent they think of competence as:

- general capacity independent on the context or specific to a given context;
- a modifiable or immutable psychological trait;
- something directly measurable or inferable;
- something transferable.

Or ask themselves whether and to what extent:

- knowledge domain is taken into account in the conceptualization of competence
- particular traits of competence are central;
- possible factors that trigger the mobilization of a competence are taken into account;
- different levels of possession of a competence are considered;
- competence is related to (high-level) performance;
- competence is extent is it definable.

The way we answer these questions reveal our understanding of the idea of competence, but it may reveal also the possible objectives for defining the idea as precisely as possible.

4.3. A provisional synthesis

Insofar we consider the competence as the prerequisite system that an individual must possess in order to perform an activity effectively, it is clear that each of the approaches described by Weinert and each of the dimensions detected by Mulder highlight relevant elements. Hence, one can raise the question about which dimension or component should be given especial value.

In my opinion, the interest in a definition of an educational construct, is that of providing a tool to organize, frame, clarify, addressing certain phenomena, situations, or problems. The question therefore arises: for what purposes is it necessary / appropriate to define the notion of competence?

Now, depending on the purposes, different conceptualizations of the idea might be appropriate or useful, different components might need to be emphasized. For instance, if one needs or wants to select people for some objectives, s/he might not need to wonder whether the components are modifiable or not, or whether they are modifiable through purposefully designed activities. But one needs to consider this issue if the idea of competence is meant for designing teaching interventions. Moreover, if the aim is to re-design curriculum or to organize teaching-learning activities, one might consider in different ways the dimension of affect. So, to me, how we conceptualize the idea of competence depends on and should be put in relation to the reasons why we think we need to do that; which phenomena, problems or situation is our elaboration of the idea competence expected to frame.

5. The Idea of Competence in Mathematics Education

In mathematics education, many constructs have been introduced that to some extent can be connected to the idea of competence: mathematical competence, mathematical literacy, mathematical proficiency, numeracy, ... Some of them were introduced before the term competence gained the current diffusion. The differences among these constructs are not only lexical ones of course; the authors who developed these constructs and coined the respective terms, did so because they felt that the existing ones did not grasp what they actually meant to.

My objective is not to make a complete overview, I will just mention some of them to illustrate some common aspects and some differences. In the next sections I will briefly outline the following constructs: mathematical habits of mind (Cuoco et al., 1996, Levasseur and Cuoco, 2003), mathematical proficiency (Kilpatrick et al., 2001), and mathematical competence (Niss, 2003, Niss and Højgaard, 2019).

5.1. Mathematical habits of mind

Cuoco et al. (1996) denounce that the mathematics, which students study and have studied for generations in high school, "has very little to do with the way mathematics is created or applied outside of school" (1996, p. 375), and contend that "[M]much more important than specific mathematical results are ***the habits of mind*** used by the people who create those results" (ibidem, p.375, my emphasis).

Habits of mind are particular ways of thinking, of facing situations, and disposing to act in the different situations. Without pretending to be exhaustive, the authors propose a repertoire of habits of mind which should be pursued in mathematics teaching. Students should become *pattern sniffers, experimenters, describers, tinkerers, inventors, visualizers, conjecturers, guessers*… Habits of mind are described and illustrates through several examples. Not all these habits are always appropriate or useful, so students should develop also an *awareness* of when to act one way or another.

To state that mathematics education should explicitly aims at fostering the development of these habits does not mean that they should be explicitly taught the same way in which content knowledge is. According to Levasseur and Cuoco (2003), these habits can and should be developed by students as they do mathematics; "the crucial element is that students be given the opportunity to develop mathematical understanding through problem solving" (p. 27). According to them, habits of mind can, therefore, be used to frame and organize mathematics curricula.

5.2. Mathematical proficiency

The construct of mathematical proficiency is elaborated in the context of a project sponsored by the US National Research Council (Kilpatrick et al., 2001). The goal of the project was to develop recommendations for teaching mathematics, teacher training and curriculum training, based on research, in order to improve the quality of learning of all students in the years from pre-kindergarten to grade 8. The problem of characterizing what can be defined as effective or successful learning of mathematics

was therefore posed within the project. Within this context, the expression *mathematical proficiency* is introduced and defined through its components (strands):

- *conceptual understanding*, which refers to the student's understanding of mathematical concepts, operations and relationships;
- *procedural fluency*, the student's ability to perform mathematical procedures in a flexible, accurate, efficient and appropriate way;
- *strategic competence*, the student's ability to formulate, represent and solve mathematical problems;
- *adaptive reasoning*, the ability to think logically and to elaborate reflections, explanations and justifications of mathematical arguments; And
- *productive disposition*, which includes the inclination to see mathematics as a sensible, useful and useful subject to learn, combined with the belief in the value of work and in one's own self-efficacy.

Among these components, both the dimension of disciplinary knowledge and the metacognitive dimension (strategic competence) are explicitly taken into account, as well as the affective dimension with reference to the personal disposition (productive disposition). All components are intertwined and interdependent: *mathematical proficiency* is not a one-dimensional trait, it cannot be achieved by focusing on just one or two of these (Kilpatrick et al., 2001, p.116). This means that the development of the conceptual understanding also feeds on the development of the other components, and vice versa.

The components of *mathematical proficiency* are identified on the basis of studies in mathematics education and cognitive psychology. In particular, with respect to the literature in mathematics education, different consonances can be recognized between the construct of mathematical proficiency and research on mathematical problem-solving (e.g. Schoenfeld, 1985, 2007).

5.3. Mathematical competence

The KOM project (Niss, 2003; Niss and Højgaard, 2011) was promoted by the Danish Ministry of Education between 2000 and 2002 with the aim of identifying on the one hand any critical elements of the Danish education system and on the other adequate tools to address these critical issues. Among the latter, the heterogeneity with which mathematics is considered and treated at different school levels is highlighted.

> "Mathematics is perceived and treated so differently at the different levels that one can hardly speak of the same subject, even if it carries the same name throughout the system […]. In other words, there are problems with the identity and coherence of mathematics as a subject across the levels" (Niss, 2003, p.3).

In this context, the notion of mathematical competence is assumed as a unifying element to be able to define what it means to "master mathematics" at all school levels, and therefore as a tool to be able to articulate the description of the curricula and the learning outcomes expected at end of each cycle of education, and to describe students"

learning progress in mathematics through different school levels. At the origin of this approach, there is therefore the common concern of defining the dimensions along which to build and organize the curriculum. To this end, part of the work of the KOM project was directed to the elaboration of the notion of mathematical competence.

> "Mathematical competence then means the ability to understand, judge, do and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role" (Niss, 2003, p. 7).

The entire framework of the KOM project has been recently revisited with the aim of updating the terminology and clarifying some definitions while keeping the overall system unchanged (Niss and Højgaard, 2019). In particular, mathematical competence is now defined as "*someone's insightful readiness to act appropriately in response to all kinds of mathematical challenges pertaining to given situations*" (ibidem, p.12). In this new formulation, the challenging nature of the situations emerges with greater clarity, at the same time it is not explicitly stated whether these situations should concern intra- or extra-mathematical contexts, or both. The term *readiness* in the definition refers only to the cognitive aspects and not to the volitional and affective ones (ibidem, p.12). In fact, the affective and volitional dimension is intentionally, deliberately excluded from the definition of mathematical competence. This decision clearly distinguishes the approach to mathematical competence taken in the KOM project from the approach to mathematical proficiency seen previously.

Mathematical competence is structured into eight components called *mathematical competencies*, which define, in a certain sense, the nature and characteristics of the mathematical actions to be undertaken to face the different challenges that may arise in different situations: *mathematical thinking competency*, *problem tackling competency, modeling competency*, *reasoning competency, representing competency, symbol and formalism competency*, *communicating competency* and *aids and tools competency*. Each of them has a dual nature: *analytical*, which consists in the ability to understand and examine aspects of mathematical activity conducted by others, and *productive*, which consists in the ability to carry out mathematical activities in the first person.

Unlike the five strands that define *mathematical proficiency*, these eight competencies, although linked to each other, are seen as independent components of *mathematical competence*. As well as *habits of mind*, these competencies are identified starting from the analysis of the characteristics of the potential action of an expert, characteristics that therefore constitute the reference term for the development of students" competence in mathematics.

5.4. A second provisional synthesis

These perspectives share a common starting point. In fact, they start, with some differences, from a feeling of unsatisfaction and an explicit critique towards the objectives of mathematics teaching as described in the curricula. Moreover, they also share common objectives, at some extent at least. To say it in Kilpatrick's words "*Competency*

frameworks are designed to demonstrate to the user that learning mathematics is more than acquiring an array of facts and that doing mathematics is more than carrying out well-rehearsed procedures" (2020, p.112).

The ability to use mathematics and mathematical skills to cope with given situations is conceptualized and described as a multi-dimensional construct constituted by different components specifically related to mathematics and mathematical activity. But the approaches differ with respect to the specific components highlighted, their nature, the relation between these components and between them and the general idea.

6. A Triadic Structure of Competence in Educational Domain

The constructs described in section 6 can be considered (at least partially) resonant with definitions coming from the field of education. Here, I report two definitions from Pellerey and Perrenoud respectiveley:

> "Competence is the ability to cope with a task or a series of tasks, to be able to initiate and orchestrate one's internal resources - cognitive, affective and volitional ones — and to use the available external ones in a coherent and fruitful way" (Pellerey, 2004, p.12, my translation).
>
> "The ability of a subject to mobilize all or part of her/his cognitive and emotional resources to deal with a family of complex situations" (Perrenoud, 1996, p.15, my translation).

These definitions share a common triadic structure: both refer explicitly to ***resources*** (including non-cognitive ones) which need to be mobilised to address some ***kinds of tasks*** or face given situations in ***effective ways***. Beyond the apparent simplicity, each of these features — resources, tasks, effectiveness — constitute a critical issue inherent to this type of conceptualization of competence, and I am tempted to state to any kind of conceptualization which could be interesting and helpful for educational purposes.

6.1. Resources

Both the definitions refer to the mobilization of internal and external resources. Though it is not explicitly stated, external resources include not only available tools, but also other human agents. With that respect, more explicit is the definition of Zarifian (1999, quoted in Terraneo and Avvanzino, 2006): "Competence is the ability to mobilize networks of actors around the same situations, to share issues, to assume areas of joint responsibility" (Zarifian, 1999, p.77, my translation).

Wittorski (1998) pushes the discourse further and suggests the possibility of attributing competence to a collective as such. That raises the issue of the relationship between individual and collective competences. This issue cannot be solved by introducing the reference to generic relational skills (also important): individuals need to develop a common image of the activity to be carried out as a whole, of the different phases and of the various individual contributions, and that it develops a specific

language to manage the interaction. Considered from this point of view, we can say that relational competence specializes in relation to a specific domain, mathematics in our case. The fact that carrying out a task requires the participation of complementary competences does not mean that these competences should or could be less developed: "Paradoxically, the stronger the collective competence, the more the individual competences become indispensable." (Terraneo and Avvanzino, 2006, p.19, my translation).

In education, shifting the focus — from the individual"s competence to the competence of a collective — raises sensitive questions both respect to individual assessment and respect to how to promote the development of individual and collective competences in a balanced way.

Beyond the individual/collective duality, Pellerey's and Perrenoud's definitions draw the attention to internal resources needed to face a given situation. Internal resources include not only cognitive resources but also metacognitive and affective ones. From point of view of mathematics education, this choice, on the one hand, recognizes and values the importance of the role of metacognitive and affective factors in shaping mathematical activity, especially in an educational context. On the other hand, it makes the whole picture even more complex, or, better, it reveals more clearly the complexity of the whole picture.

Internal resources, be they cognitive, volitional or affective, depend largely on the specific domain evoked by the situation. Thus, there remain the issues to identify the resources needed to carry out mathematics activities efficiently, to suitably characterize them; there remain the issues whether it is possible to set the development of these resources be explicit educational goals, and in case how to intentionally promote them. The research studies discussed in section 6 address and frame these issues in different ways.

6.2. Tasks

Competence is defined as the ability to cope with a task or a family of tasks. But, what is exactly meant by task?

Duncker (1935) defines a problem as something that arises when a living being has a goal but does not know how to reach it. Drawing on this definition, Zan (2007) proposes to distinguish between problem and task — depending on the existence or not of a goal and of its sense for the individual — and between problem and exercise — depending on whether the solver has or does not have a procedure available to reach the goal. In this same line of thought, Terraneo and Avvanzino (2006) propose to distinguish, in the context of work psychology, between prescribed and actual task, between explicit prescription, implicit prescription and perceived prescription. It is a type of distinction that is absolutely pertinent to the educational field, too.

Another important issue concerns the distinction between real-life or in-context tasks and simulated or contextualized tasks; with the recognized potential but also the limits of proposing simulated tasks in the classroom (Mulder et al., 2007; Palm, 2002).

Many conceptualizations of the idea of competence stress the challenging nature of tasks. This necessarily leads to relativize the idea of competence, to problematize the performance as an indicator of the presence or absence of a competence, to recognize the importance of devoting a specific attention to the affective dimension.

Finally, one speaks of family of tasks, tasks sharing analogies, but what does it mean? Who is expected to see these analogies, whose point of view is assumed? For instance, one could consider a given *field of experience*, that is as a field of human cultural experience recognized as homogeneous and unitary (Boero et al., 1995). Or, one could consider a family of tasks, as a set of situations whose mastery requires a certain system of concepts, procedures and symbolic representations strictly connected between them, that is as given *conceptual field* (Vergnaud, 1995). The organizing principles assumed in the two evoked perspectives are quite different, and this difference is not without implications form an educational point of view.

Finally, let's note that the descriptions of tasks and resources are strictly connected, in fact the more clearly the "family of tasks" at stake, or the "domain of competence", can be defined, the easier is the analysis of the resources to be developed to operate effectively in that domain. At the same time, we can say that a task can belong to the "domains" of different competences and require the mobilization of different competences.

6.3. Efficiency

Finally in the definitions of competence we are examining, there is an indirect reference to the theme of assessment, as the question arises of what "fruitful and effective mobilization of resources" means, and how it is assessed. This leads to the necessity of considering who assesses, why, for what purposes, what is assessed and how.

When an individual faces a certain situation, the individual her/himself, an expert, a collective (of possible non-experts) can, each, be involved in the evaluation of the action of the individual, of the outcome of the mobilization and orchestration of her/his resources, of its eventual fecundity and efficacy. Each of these agents can have different criteria for the assessment, focuses on different possible indicators. The situation itself can provide, or not, feedback, which can used for the assessment.

There might be several reasons why it can be necessary to establish whether the individual succeeds to cope with the situation in an effective way. For instance, the task can be relevant in itself and needed to be solved. The assessment of how the task has been accomplished can be needed for devise future initiatives depending on the accomplishment of the task itself. In some contexts, assessment is made for selecting people, for certifying competences, for promoting learning.

With that respect it is worthwhile noticing that if one considers an educational context, two dimensions are simultaneously present:

- The assessment of the accomplishment of the task per se (which can be made by different agents)

- The assessment of the accomplishment of the task as an indicator of competence, which is made by an evaluator (basically the teacher).

With respect to the issue of the assessment in schools, it seems to me that the discourse is backlog: one pretends to evaluate competences after a teaching practice that is not aimed at developing them. It seems to me that Mulder and colleague's criticism of some vocational training programs should sound like a warning to school: "the emphasis on competence assessment is unbalanced, and [that] it frustrates learning and development more than it supports it" (Mulder et al., 2007).

7. Conclusions

The discussion developed does not solve the problem of the definition of mathematical competence and of its elusiveness. This was not the objective. Rather, I meant to discuss the complexity of the idea of competence and to put the issue of its possible characterization it in a different perspective: the choice of an approach to the definition of competence brings into play objectives and systems of values; therefore, the first step in being able to start a fruitful discussion on this issue, at any level, is to explain them.

Competence, as we have seen, is a multi-faceted complex idea. It emerges, in any conceptualization, as a ***multidimensional construct***. By stating that competence is a *construct*, I mean to highlight its artefact-like nature: it is a conceptual construction which can be useful to frame, organize clarify, complex phenomena. In this sense it is not an attribute of an individual but express the point of view of someone on something. To me the key-issue is what idea we mean to capture through this notion and for what purposes we need/want to capture this idea.

It is *multidimensional*: as we have seen, any conceptualization refers to "several distinct but related dimensions or components treated as a single theoretical concept" (Edwards, 2001, p. 144). It needs to be rich to capture complex phenomena but needs no to be too complex for being useful. Diverse components can be considered with different emphasis; the relation amongst these components and between them and the more general idea need to be clarified. Since we need the breadth and comprehensiveness of a multidimensional construct and the precision and clarity of its single dimensions.

Personally, some of the reasons for which I began to investigate this notion are still there, for instance: how can one foster the development of students" mathematical competence? What does it mean? How can one attest the development of competences?

We need to make the notion of mathematical competence more operational, to define the unit of analysis in educational research on mathematical competence and problematize how we can capitalize on the existing research results in mathematics education concerning many aspects which can be related to the idea of competence.

References

P. Boero, C. Dapueto, P. Ferrari, E. Ferrero, R. Garuti, E. Lemut, L. Parenti, and E. Scali (1995). Aspects of the Mathematics-Culture Relationship in mathematics teaching learning in compulsory school. *Proc. 19th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 1, pp. 151–166). Recife, Brasil: PME.

A. Cuoco, E. P. Goldenberg, and J. Mark (1996). Habits of Mind: An Organizing Principle for Mathematics Curricula. *Journal of Mathematical Behavior,* 15, 375–40.

J. Dolz and E. Ollagnier (2002). *L'énigme de la compétence en éducation*. Louvain-la-Neuve: De Boeck Supérieur.

K. Duncker (1935). *Zur Psychologie des Produktiven Denkens*. Berlin: Springer.

J. R. Edwards (2001). Multidimensional Constructs in Organizational Behavior Research: An Integrative Analytical Framework. *Organizational Research Methods,* 4(2), 144–192

P. Gilbert and M. Parlier (1992). La compétence: Du «mot-valise» au concept opératoire. *Actualité de la Formation Permanente,* 116, 14–18.

J. Kilpatrick, J. Swafford, and B. Findell (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

J. Kilpatrick (2001). Understanding mathematical literacy: the contribution of research. *Educational Studies in Mathematics*, 47, 101–116.

J. Kilpatrick (2020). Competency Frameworks in Mathematics Education. In S. Lerman (ed), *Encyclopedia of Mathematics Education*. Dordrecht, Heidelberg, New York, London: Springer, pp. 110–113.

K. Levasseur and A. Cuoco (2003). Mathematical habits of mind. In H. L. Schoen (Ed.), *Teaching Mathematics through Problem Solving: Grade 6–12*. Reston, VA: National Council of Teachers of Mathematics. pp. 23–37.

A. Marzano and I. S. Iannotta (2015). Authentic evaluation of competence. *The Online Journal of Quality in Higher Education,* 2(2), 9–16.

M. Mulder (2007). Competence — the essence and use of the concept in ICVT. *European Journal of Vocational Training,* 40(1), 5–21.

M. Mulder (2017). Competence Theory and Research: a synthesis. In: M. Mulder (Ed.), *Competence-Based Vocational and Professional Education. Bridging the Worlds of Work and Education*. Cham, Switzerland: Springer, pp. 1071–106.

M. Mulder, T. Weigel, and K. Collins (2007). The concept of competence in the development of vocational education and training in selected EU member states. *A critical analysis. Journal of Vocational Education and Training,* 59(1), 65–85.

M. Niss (2003). Mathematical competencies and the learning of mathematics: the Danish KOM project. In: A. Gagatsis and S. Papastavridis (eds). *3° Mediterranean Conference on Mathematical Education, 3–5 January 2003.* Athens: Hellenic Mathematical Society, pp. 115–124.

M. Niss and T. Højgaard (2019). Mathematical competencies revisited. *Educational Studies in Mathematics,* 102, 9–28.

T. Palm (2002). *The realism of mathematical school tasks — Features and consequences*. Doctoral Thesis No 24. Umeå University: Umeå, Sweden.

M. Pellerey (2004). *Le competenze individuali e il Portfolio*. Firenze: La Nuova Italia.

M. Pellerey (2013). Le competenze strategiche: loro natura, sviluppo e valutazione (Seconda Parte). *Orientamenti Pedagogici,* 60(2), 479–497.

P. Perrenoud (1996). *Enseigner, agir dans l'urgence décider dans l'incertitude*. Paris: ESF Editeur.

A. H. Schoenfeld (1985). *Mathematical Problem Solving*. New York: Academic Press.

A. H. Schoenfeld (2007). What is mathematical proficiency and how can it be assessed? In A. H. Schoenfeld (Ed.), *Assessing Mathematical Proficiency*. Mathematical Sciences Research Institute Publications, Vol. 53. New York, NY: Cambridge University Press, pp. 9–73.

F. Terraneo and N. Avvanzino (2006). Le concept de compétence en regard de l'évolution du travail: définitions et perspectives. *Recherche en Soins Infirmiers*, 87, pp. 16–24.

G. Vergnaud (1990). La théorie des champs conceptuels. *Recherches en Didactique des Matthematiques*, 10, 133–169.

F. E. Weinert (2001). Concept of competence: a conceptual clarification. In: D.S.Rychen and L.H. Salganik (eds). *Defining and Selecting Key Competencies*. Seattle: Hogrefe & Huber, pp. 45–65.

R. Wittorski (1998). De la fabrication des compétences. *Education Permanente*, 135, 57−70.

R. Zan (2007). *DifficoltÀ In Matematica. Osservare, Interpretare, Intervenire*. Springer-Verlag Italia.

P. Zarifian (1999). *Objectif Competence*. Paris: Editions Liaisons.